Dynamic Screening in Thermonuclear Reactions

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ABSTRACT

It has recently been argued that there are no dynamic screening corrections to Salpeter's enhancement factor in thermonuclear reactions, in the weak-screening limit. The arguments used were: 1) The Gibbs probability distribution is factorable into two parts, one of which, $exp(-\beta \sum e_i e_j / r_{ij})$ $(\beta = 1/k_B T)$, is independent of velocity space; and 2) The enhancement factor is $w = 1 + \beta^2 e^2 Z_1 Z_2 \langle \phi^2 \rangle$ with $\langle \phi^2 \rangle_k = \langle E^2 \rangle_k / k^2$ and $\langle E^2 \rangle_k / (8\pi) = (T/2)[1 - \varepsilon^{-1}(0,k)]$. We show that both of these arguments are incorrect.

Subject headings: nuclear reactions, nucleosynthesis, abundances

1. Introduction

Our knowledge of reaction rates in the Sun is becoming more and more accurate. There have been several works dealing with electrostatic screening effects in the solar plasma (as for example, Carraro, Schäfer, & Koonin 1988; Gruzinov & Bahcall 1997; Brüggen & Gough 1997). Recently, more precise calculations have been made beyond the linear regime (Gruzinov & Bahcall 1998).

The screening has the effect of lowering the Coulomb barrier between the interacting ions, therefore enhancing the reaction rates. This is included in the enhancing factor in the weak screening limit (where $Z_1 Z_2 e^2 / R_D T \ll 1$ and R_D is the Debye radius)

$$w = exp\ \lambda\ , \tag{1}$$

where

$$\lambda = Z_1 Z_2 e^2 / R_D T\ . \tag{2}$$

This screeening factor uses the Debye-Hückel expression.

Usually, the calculation is made in the electrostatic case, based on the calculation of Salpeter (1954). It is assumed that the motion of the screened ion is slow, compared to the motion of the screening particles. Carraro, Schäfer, & Koonin (1988) have studied the case of dynamic screening.

In a recent study, Gruzinov (1998) argued that in the weak screening limit, there is no dynamic screening corrections to Salpeter's enhancement factor, even for high energies. He based his conclusion on two arguments: 1) For thermodynamic equilibrium, the Gibbs probability distributions in velocity and configuration space are decoupled; and 2) Through an analysis of the thermal electric field, we can estimate the random electrostatic potential and conclude that the enhancement of the reaction rates is given by the Salpeter expression.

We show below that although these arguments are simple, a careful examination shows that they are wrong. In §2 and §3 we recall the arguments and show why they are incorrect.

2. Gibbs Distribution

The enhancement factor for a reaction rate between nuclei of charges $Z_1 e$ and $Z_2 e$ is

$$w = exp(-Z_2 e \phi_0 / T) \tag{3}$$

($k_B = 1$), where ϕ_0 is the electrostatic potential, created by the plasma on the particle $Z_1 e$.

A test particle $Z_1 e$ moving through a plasma with velocity v' suffers dynamic screening. The electrostatic potential is written as (see Krall & Trivelpiece (1973), chap.11)

$$\phi_0 = 4\pi e Z_1 \int \frac{d^3k}{(2\pi)^3} \left[\frac{1}{\varepsilon(k, kv')} - 1 \right] k^{-2} \ , \tag{4}$$

where ε is the dielectric permittivity, which describes the plasma response to the test particle. (ε depends on the velocity distribution of the plasma particles $f(v)$, which usually is taken as Maxwellian). From the above expression, it can be seen that ϕ_0 depends on the velocity v' of particle $Z_1 e$. Gruzinov (1998) argued that the Gibbs probability distribution ρ is

$$\rho \sim exp \left(-\beta \sum \frac{m_i v_i^2}{2} - \beta \sum \frac{e_i e_j}{r_{ij}} \right) \ , \tag{5}$$

(where $\beta = 1/T$) which can be factorable into

$$\rho \sim exp \left(-\beta \sum \frac{m_i v_i^2}{2} \right) exp \left(-\beta \sum \frac{e_i e_j}{r_{ij}} \right) \ . \tag{6}$$

From the above, it was argued that the distributions in velocity and configuration space are decoupled.

This first argument is simple. However, it is based on a misunderstanding. The general Gibbs probability distribution ρ for a plasma is

$$\rho \sim exp\left(-\beta \sum \frac{m_i v_i^2}{2}\right) exp\left(-\beta \sum_{i>k} \sum_k W_{ik}\right) \ , \tag{7}$$

where W_{ik} is the interaction energy between the particles. W_{ik} is the interaction energy of particle i with all the other particles in the plasma. This energy is the Coulomb energy $e_i e_j / r_{ij}$, related to the positions of all the other particles. It is an assumption that their positions r_i, r_j are independent of their velocities. However, we know from dynamic screening, Eq. (4), that their coordinates are, in fact, velocity dependent. Eq. (6) states that the coordinates are independent of the velocities only in zero order. The exact Gibbs distribution takes into account dynamic corrections.

In fact, this argument is circular. The separability of the Gibbs distribution in velocity and configuration contributions is true only if the particles are moving under a (static) conservative force. The argument to show that there are no dynamical contributions, therefore, rests on a statement that is valid only if there are no dynamical contributions, which, in fact, is what he set out to prove.

3. Thermal Electric Field

Due to thermal fluctuations in the plasma, the reaction rate between two fast moving ions $Z_1 e$ and $Z_2 e$ is enhanced. The enhancement factor of a reaction rate is

$$w = 1 + \beta^2 e^2 Z_1 Z_2 \langle \phi^2 \rangle \ , \tag{8}$$

where $\langle \phi^2 \rangle$ is the average of the square of the random electrostatic potential ϕ. $\langle \phi^2 \rangle$ is given by

$$\langle \phi^2 \rangle = \int \frac{d^3 k}{(2\pi)^3} \langle \phi^2 \rangle_k \ , \tag{9}$$

where $\langle\phi^2\rangle_k = \langle E^2\rangle_k / k^2$.

In his second argument, the expression used for the fluctuation electric field was (Krall & Trivelpice (1973), chap.11)

$$\frac{\langle E^2\rangle_k}{(8\pi)} = \left(\frac{T}{2}\right)\left[1 - \frac{1}{\varepsilon(0,k)}\right] \ . \tag{10}$$

Substituing this expression in Eq.(8) and Eq.(9), Salpeter's expression is obtained.

However, the expression of the thermal electric field (for a Maxwellian plasma), given by Eq. (10), assumes that $\omega \ll T$ ($\hbar, k_B = 1$). The general expression for the intensity of the electric field is given by the *Fluctuation-Dissipation Theorem* (see for example Sitenko (1967), Akhiezer et al. (1975)):

$$\frac{\langle E^2\rangle_{k\omega}}{(8\pi)} = \frac{1}{e^{\omega/T} - 1}\frac{Im\varepsilon}{\mid\varepsilon\mid^2} \tag{11}$$

and

$$\frac{\langle E^2\rangle_k}{(8\pi)} = \int d\omega \frac{1}{e^{\omega/T} - 1}\frac{Im\varepsilon}{\mid\varepsilon\mid^2} \ . \tag{12}$$

This expression includes electrostatic Langmuir waves as well as all other fluctuations that exist in a plasma. In the limit of $\omega \ll T$,

$$\frac{\langle E^2\rangle_k}{(8\pi)} = \int d\omega \frac{T}{\omega}\frac{Im\varepsilon}{\mid\varepsilon\mid^2} \ , \tag{13}$$

for which it is then possible to use the Kramers-Kronig relations. Eq.(13) then turns out to be

$$\frac{\langle E^2\rangle_k}{(8\pi)} = \left(\frac{T}{2}\right)\left[1 - \frac{1}{\varepsilon(0,k)}\right] \ , \tag{14}$$

which is the expression used.

The assumption that $\omega \ll T$, however, is very strong. In fact, in the case of the transverse electric field, we showed (Opher & Opher (1997a, 1997b)) that only by not making this strong assumption, is the blackbody at high frequencies

obtained. We recently showed (Opher & Opher (1999)), that by not making the assumption that $\omega \ll T$, the energy of a plasma in the classical limit is larger than previously thought.

Without assuming that $\omega \ll T$, the enhancement factor is given by Eq. (8) with Eq. (12).

It is also to be noted that the second argument is also circular. It assumes that $\omega \ll T$, making it a static analysis, which is then used to prove that there does not exist a dynamic contribution.

It is to be emphasized that we are not arguing here whether or not dynamic screening exists in thermonuclear reactions. We only show that the arguments used by Gruzinov (1998), to prove that dynamic screening does not exist, are not valid.

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